Assignment 0 is for your own edification, it should provide some problems for you to refresh/test/hone your Matlab programming. This assignment will not be assessed — you do not need to submit your answers.

**Question 1**
Find the angle, $\theta$, between the vectors, $v_1 = (1.5, -2, 4, 10)$ and $v_2 = (3.1, -1, 2, 2.5)$, using the formula
$$\cos \theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|}.$$ 

**Question 2**
Evaluate and plot the Chebyshev polynomial of degree 5 at 100 evenly spaced points in the interval $x \in [-1, 1]$. Use the fact that we have the following recurrence relation for Chebyshev polynomials (where $T_k$ denotes the Chebyshev polynomial of degree $k$):
$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x), \quad \text{for } k \geq 2,$$ 
and $T_0(x) = 1$, $T_1(x) = x$.
Plot $T_3(x)T_5(y)$ on a 100x100 grid on the domain $(x,y) \in [-1, 1]^2$. Try different plotting functions in Matlab for this grid-based data e.g. `mesh`, `surf`, `contour`.

**Question 3**
Use the iteration
$$x_{k+1} = \frac{1}{2} \left( x_k + \frac{a}{x_k} \right).$$
to approximate $\sqrt{a}$. (This is known as Heron’s formula\(^1\) and in fact it is equivalent to Newton’s method for $f(x) = x^2 - a$.) Choose an “initial guess” $x_0 = a$ and iterate until $|x_{k+1} - x_k| < \text{TOL}$. Determine the number of iterations required to compute $\sqrt{5}$ in the cases TOL = $10^{-3}$ and TOL = $10^{-9}$.

**Question 4**
In this question we examine the behavior of a finite difference approximation as $h \to 0$. (Aside: If $y = \alpha h^\beta$, then $\log(y) = \log(\alpha) + \beta \log(h)$. As a result, log-log axes are often the clearest way to show convergence results since if the error behaves like $O(h^\beta)$, then on log-log axes we will see a straight line with gradient $\beta$.]
Let $f(x) = \tan(x)$, and consider the second order finite difference approximation
$$f_{\text{diff},2}(x;h) \equiv \frac{f(x+h) - f(x-h)}{2h}.$$ 
Plot the relative error in the $f_{\text{diff},2}(x;h)$ approximation at $x = 1$ as a function of $h$ for $h = 10^{-k}, k \in \{1, 1.5, 2, \ldots, 15.5, 16\}$. Make sure you use a “log-log” plot (use the command `loglog` in Matlab).

\(^1\)Heron of Alexandria, 10-70 AD.
Overlay dashed lines $\alpha_1 h^2$ and $\alpha_2/h$ on the same log-log axes to compare to the error plot from above. (You should choose $\alpha_1, \alpha_2$ so that $\alpha_1 h^2$ and $\alpha_2/h$ are “close to” the error plot from above so that it is easy to see that the lines are aligned for certain ranges of $h$.)

**Question 5**

$y = \sin(x)$ is an analytic function, which means that the Taylor series

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

converges for any $x \in \mathbb{R}$. Write a Matlab function (call it `sinTaylorSeries`) to evaluate this series; the function should take arguments $x$ (a vector of values to evaluate the function at) and $N$ (the number of terms in the series) and should return a vector $y$ of function values and a vector $\text{err}$ of (absolute) error values with respect to Matlab’s built-in $\sin(x)$ function.

Use your function to plot $y$ and $|\text{err}|$ on the intervals $[-\pi, \pi]$ and $[-10\pi, 10\pi]$ for $N = 10$ and $N = 100$. (Use a “semilog-y” plot for $|\text{err}|$, i.e. `semilogy` in Matlab.)